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Modified Estimation Using Different Linear Combination

Shabnum Gul*, P. P. Singh, Nisar Ahmad Khan

J. S. University Shikhibad Farozabad U. P Baramulla (193101)

*Corresponding Author E-mail: khalidzara2018@gmail.com

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ABSTRACT

The present study was taken under consideration in order to propose new modified estimators using different linear combinations for population mean using the auxiliary information of Mid-Range with coefficient of correlation, coefficient of variation and coefficient of skewness in order to achieve more precision in estimates than the already existing estimators. The properties associated with the proposed estimators are assessed by mean square error and bias and compared with the existing estimators. In the support of the theoretical proposed work we have given numerical illustration.

Keywords: *Auxiliary information, Linear combinations estimators, Mean square error, Bias, Efficiency.*

INTRODUCTION

An important purpose of sampling theory is to make sampling more efficient. It attempts to develop methods of sample selection and of estimation that provides, at the lowest possible cost, estimates that are precise enough for our purpose. This principle of specified precision at minimum cost recurs repeatedly in the presentation of theory. Estimation theory is an important part of statistical studies, whereby, population parameters are obtained using sample statistics. In any survey work, the experimenter's interest is to make use of methods that will improve precisions of estimates of the population parameters both at the design stage and estimation stage. These parameters can be totals, means or proportions of some desired characters. In sample surveys, auxiliary information is used at selection as well as estimation stages to improve the design as well as obtaining more efficient estimators. Increased precision can be obtained when study variable Y is highly correlated with auxiliary variable X. Usually, in a class of efficient estimators, the estimator with minimum variance or mean square error is regarded as the most efficient estimator. Linear combination estimators are good examples in this context. Cochran (1940) initiated the use of auxiliary information at estimation stage and proposed ratio estimator for population mean. It is well established fact that linear combination estimators provide better efficiency in comparison to simple mean estimator if the study variable and auxiliary variable are positively correlated.

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If the correlation between the study variable and auxiliary variables negative, product estimator given by Robson (1957) is more efficient than simple mean estimator.

Further improvements are also achieved on the classical linear combination by introducing a large number of modified linear combinatiob with the use of known parameters like, coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion one may refer to Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Murthy (1967), Prasad (1989), Rao (1991), Singh (2003), Singh and Tailor (2003, 2005), Singh et al (2004), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Yan and Tian (2010).

Further, had taken initiative by proposed modified linear combinations estimator for estimating the population mean of the study variable by using the population median of the auxiliary variable.

The objective of the paper is to propose modified estimators for estimating the population mean by using their linear combinations with the correlation coefficient and the coefficient of skewness of the auxiliary variable.

2. Notations Used

The following are the notations used in the paper:

N Population n Sample size

$f = n/N$ Sampling fraction Y Study variable

X Auxiliary variable $\bar{X}, \bar{Y}$ Population mean

$\bar{x}, \bar{y}$ Sample means x, y Sample totals

s_x, s_y Population standard deviations s_{xy} Population covariance between c_x, c_y Coefficient of variation ρ Correlation coefficient

$B(.)$ Bias of the Estimator $MSE(.)$ Mean square error of the estimator $MR = \dfrac{X_{(1)} + X_{(N)}}{2}$ Mid-range β_2 Kurtosis β_1 Skewness

3. Procedure and Definitions

Let $U = \{U_1, U_2, U_3, ..., U_N\}$ be a finite population of N distinct and identifiable units. Let y and x denotes the study variable and the auxiliary variable taking values y_i and x_i respectively on the i[th]unit (i = 1, 2,…, N). For estimating the population mean $\bar{Y}$ of y a simple random sample of size n is drawn without replacement from the population U. Then the classical ratio estimator is defined.

$$t_R = \frac{\bar{y}}{\bar{x}} \bar{X}\,;\ if\ \ \bar{x} \neq 0$$

Where $\bar{X}$, the population mean of the auxiliary variable x is known.

The mean square error expressions of the ratio and product estimators are

$$MSE(\bar{y}_{\mathrm{k_e}}) = \frac{\sigma_y^2}{\mathrm{n}} - \frac{2\bar{Y}^2 C_y^4 + C_x}{n(4C_y^2 + C_x + 1)}$$

Further, a list of modified ratio estimators is given in table 1 is used for assessing the performance of the proposed estimator along with their bias and mean squared error expressions.

Table 1: Existing modified ratio type estimators with their biases and mean squared errors

Estimators	Bias, B (.)	Mean square error, MSE (.)	Constants θ_i or R_i
$t0 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi (2004)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_0^2$	$\left(\frac{1-f}{n}\right)(R_0^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R0=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$t_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1)$ Yan and Tian (2010)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_1^2$	$\left(\frac{1-f}{n}\right)(R_1^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_1=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$t_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\beta_2)}(\bar{X}\beta_1+\beta_2)$ Yan and Tian (2010)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_2^2$	$\left(\frac{1-f}{n}\right)(R_2^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_2=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$t_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho_{yx})}(\bar{X}+\rho_{yx})$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_3^2$	$\left(\frac{1-f}{n}\right)(R_3^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_3=\frac{\bar{Y}}{\bar{X}+\rho_{yx}}$
$t_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho_{yx})}(\bar{X}C_x+\rho_{yx})$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_4^2$	$\left(\frac{1-f}{n}\right)(R_4^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_4=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho_{yx}}$
$t_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho_{yx}+C_x)}(\bar{X}\rho_{yx}+C_x)$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_5^2$	$\left(\frac{1-f}{n}\right)(R_5^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_5=\frac{\bar{Y}\rho_{yx}}{\bar{X}\rho_{yx}+C_x}$
$t_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho_{yx}+\beta_2)}(\bar{X}\rho_{yx}+\beta_2)$ Kadilar and Cingi (2006)	$\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{Y}}R_6^2$	$\left(\frac{1-f}{n}\right)(R_6^2S_x^2+S_y^2(1-\rho_{yx}^2))$	$R_6=\frac{\bar{Y}\rho_{yx}}{\bar{X}\rho_{yx}+\beta_2}$

4. Proposed Estimator

we have proposed an estimator for estimating the population mean when the mid-range, correlation coefficient and coefficient of skewness of the auxiliary variable is known.

$$t_{sts1} = \bar{y}\left\{\alpha\left(\frac{\bar{X}+MR}{\bar{x}+MR}\right)+(1-\alpha)\left(\frac{\bar{x}+MR}{\bar{X}+MR}\right)\right\}$$

$$t_{sts2} = \bar{y}\left\{\alpha\left(\frac{\bar{X}\rho+MR}{\bar{x}\rho+MR}\right)+(1-\alpha)\left(\frac{\bar{x}\rho+MR}{\bar{X}\rho+MR}\right)\right\}$$

$$t_{sts3} = \bar{y}\left\{\alpha\left(\frac{\bar{X}\beta_1+MR}{\bar{x}\beta_1+MR}\right)+(1-\alpha)\left(\frac{\bar{x}\beta_1+MR}{\bar{X}\beta_1+MR}\right)\right\}$$

Where MR is the Mid-range of the auxiliary variable X.

To the first degree of approximation, we have obtained the expression of bias and mean squared error (MSE) of the proposed estimator as

$$\text{Bias } (\bar{y}_k 1) = \text{E}(\bar{y}_k - \bar{Y}) = \frac{k(2k+1)}{n}\left(\frac{\mu_2}{\bar{Y}} - Y_1\sigma_y\right) + \frac{\bar{Y}}{2n}\,[k(k+1)(\beta_2-1)] + \beta 1$$

$$\text{Bias } (\bar{y}_k 2) = \text{E}(\bar{y}_k - \bar{Y}) = \frac{k(2k+1)}{n}\left(\frac{\mu_2}{\bar{Y}} - Y_1\sigma_y\right) + \frac{\bar{Y}}{2n}\,[k(k+1)(\beta_2-1)] + \beta 2$$

$$\text{Bias }(\bar{y}_k 3) = \text{E}\left(\bar{y}_k - \bar{Y}\right) = \frac{k(2k+1)}{n}\left(\frac{\mu_2}{\bar{\bar{Y}}} - Y_1\sigma_y\right) + \frac{\bar{Y}}{2n}\left[k(k+1)(\beta_2 - 1)\right] + \beta 3$$

And

$$MSE(\bar{y}_k) = \frac{\sigma_y^2}{n} - \frac{\bar{Y}^2(2C_y^2 + C_x - y_1C_y + C_x)^2}{n\{4C_y^2 + C_x + (\beta_2 - 1) - 4y_1C_y + C_x\}} + \phi 1_{+\alpha 01}$$

$$MSE(\bar{y}_k) = \frac{\sigma_y^2}{n} - \frac{\bar{Y}^2(2C_y^2 + C_x - y_1C_y + C_x)^2}{n\{4C_y^2 + C_x + (\beta_2 - 1) - 4y_1C_y + C_x\}} + \phi 2 + \alpha 02$$

$$MSE(\bar{y}_k) = \frac{\sigma_y^2}{n} - \frac{\bar{Y}^2(2C_y^2 + C_x - y_1C_y + C_x)^2}{n\{4C_y^2 + C_x + (\beta_2 - 1) - 4y_1C_y + C_x\}} + \phi 3 + \alpha 03$$

Where $\phi_1 = \dfrac{\bar{X}}{\bar{X} + MR}$, $\phi_2 = \dfrac{\bar{X}\rho}{\bar{X}\rho + MR}$ and $\phi_3 = \dfrac{\bar{X}\beta_1}{\bar{X}\beta_1 + MR}$,

The, MSE will be minimum when

$$\alpha = \frac{1}{2}\left(1 - \frac{\rho_{yx}C_y}{\phi_1 C_x}\right) = \alpha_{01}\,(say),\ \alpha = \frac{1}{2}\left(1 - \frac{\rho_{yx}C_y}{\phi_2 C_x}\right) = \alpha_{02}\,(say)$$

and $\alpha = \dfrac{1}{2}\left(1 - \dfrac{\rho_{yx}C_y}{\phi_3 C_x}\right) = \alpha_{03}\,(say)$,

by substituting the minimum value of α in the proposed estimators, one can get the asymptotically optimum estimators (AOE) as

$$t_{sts1}(opt) = \bar{y}\left\{\alpha_{01}\left(\frac{\bar{X} + MR}{\bar{x} + MR}\right) + (1 - \alpha_{01})\left(\frac{\bar{x} + MR}{\bar{X} + MR}\right)\right\}$$

$$t_{sts2}(opt) = \bar{y}\left\{\alpha_{02}\left(\frac{\bar{X}\rho + MR}{\bar{x}\rho + MR}\right) + (1 - \alpha_{02})\left(\frac{\bar{x}\rho + MR}{\bar{X}\rho + MR}\right)\right\}$$

$$t_{sts3}(opt) = \bar{y}\left\{\alpha_{03}\left(\frac{\bar{X}\beta_1 + MR}{\bar{x}\beta_1 + MR}\right) + (1 - \alpha_{03})\left(\frac{\bar{x}\beta_1 + MR}{\bar{X}\beta_1 + MR}\right)\right\}$$

Thus the optimum MSE of, t_{sts1}, t_{sts2} and t_{sts3} is

$$MSE(\bar{y}_{\mathrm{k_e}}) = \frac{\sigma_y^2}{\mathrm{n}} - \frac{2\bar{Y}^2 C_y^4 + C_x}{n\left(4C_y^2 + C_x + 1\right)}$$

5. Efficiency Comparison

For comparison of proposed estimator with the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as

$$Min.MSE(t_{sts1}, t_{sts2}, t_{sts3}) \le MSE(t_i; i = 1,2,3,4,5)\ if\ \ \rho_{yx} \le \frac{\theta_i C_x}{C_y}$$

$$Min.MSE(t_{sts1}, t_{sts2}, t_{sts3}) \le MSE(t_i; i = 6,...,15)\ if\ \ R_i^2 S_x^2 \ge 0$$

$$Min.MSE(t_{sts1}, t_{sts2}, t_{sts3}) \le MSE(t_{16})\ if\ \ \rho_{yx} \le \frac{C_x}{C_y}.$$

From the above conditions, it is noted that the proposed estimators are more efficient among other discussed estimators if the above conditions holds true.

6. Empirical Study

To demonstrate the performance of the suggested estimator empirically in comparison to other estimators. We have used the Murthy (1967) where Y is output for 34 factories in a region and X is Data on number of workers. The descriptions of the population are given below.

Table 2: Characteristics of the Population

Population (Murthy 1967)	
$N = 34$	$C_y = 0.8561$
$n = 20$	$S_x = 150.2150$
$\bar{Y} = 856.4117$	$C_x = 0.7531$
$\bar{X} = 199.4412$	$\beta_2 = 1.0445$
$\rho = 0.4453$	$\beta_1 = 1.1823$
$S_y = 733.1407$	$MR = 320$
$M_d = 1.48$	$QD = 80.25$

Here, we have computed mean squared error (MSE) and the Bias of the estimators.

The results are given in the following table.

Table 3: The mean squared errors and Bias of the existing and proposed estimators

Estimators	**MSE**	**Bias**
T_0; Kadilar and Cingi (2004)	2.317192	0.974223
T1;Yan and Tian (2010)	60.5325	14.60269
T_2; Yan and Tian (2010)	35.18871	10.99178
T_3; Kadilar and Cingi (2006)	53.98248	13.76056
T4; Kadilar and Cingi (2006)	52.63652	13.58105
T5; Kadilar and Cingi (2006)	50.78761	13.33051
T6; Kadilar and Cingi (2006)	42.40512	12.12991
Proposed 1	**0.855269351**	**0.332265112**
Proposed 2	**0.855269334**	**0.312863365**
Proposed 3	**0.855269328**	**0.257737132**

CONCLUSION

In this paper we have proposed modified estimators based on simple random sampling without replacement by using the auxiliary variable, under the situation when mid-range, coefficient of skewness and correlation coefficient is known. We found that the performances of our proposed estimators in terms of mean square error are more efficient than all other existing estimators in the literature. Hence we strongly recommend that our proposed estimators preferred over the existing estimators for use in practical application.

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